

DEFINITIONS

- A grammar is said to be **left-recursive** if it has a non-terminal A s.t. there is a derivation $A \Rightarrow A\alpha$ for some string α of terminals and non-terminals.
- **Immediate** or **simple** left-recursion is when the grammar contains a production of the form $A \rightarrow A\alpha$ for some string α of terminals and non-terminals.

ELIMINATING LEFT RECURSION

- Eliminating simple left-recursion:
 - $S \rightarrow S\alpha \mid \beta$ where $\alpha, \beta \in (N \cup T)^*$ but don't start with S
 - Language generated is $\beta\alpha^*$ (0 or more α 's) = $\beta\alpha^*$
 - Replace by

$$S \rightarrow \beta S'$$

$$S' \rightarrow \alpha S' \mid \epsilon$$
- More complex situation:
 - $S \rightarrow S\alpha_1 \mid \dots \mid S\alpha_n \mid \beta_1 \mid \dots \mid \beta_m$
 where α 's, β 's $\in (N \cup T)^*$ but don't start with S
 - Replace by:

$$S \rightarrow BS'$$

$$B \rightarrow \beta_1 \mid \dots \mid \beta_m$$

$$S' \rightarrow \epsilon \mid \alpha_1 S' \mid \dots \mid \alpha_n S'$$
- Eliminating general left recursion:
 - Make a list of all non-terminals in an order
 - For each non-terminal
 - Examine all its productions. If the right-hand side begins with a non-terminal earlier in the list (e.g. $B \rightarrow A\beta$)
 1. Try to reorder B before A if it solves the problem,
 2. Otherwise look at the earlier productions.
 - If these are: $A \rightarrow \alpha_1 \mid \dots \mid \alpha_n$
 - Substitute $B \rightarrow \alpha_1\beta \mid \dots \mid \alpha_n\beta$
 - Continue until no right-hand side begins with a non-terminal earlier in the list.
 - Remove the immediate left recursions (if any) for this non-terminal